

REPORT DOCUMENTATION PAGE				Form Approved OMB No. 0704-0188	
<p>The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.</p> <p><b>PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.</b></p>					
1. REPORT DATE (DD-MM-YYYY) MAY 2013		2. REPORT TYPE JOURNAL ARTICLE (Post Print)		3. DATES COVERED (From - To) JUN 2010 – FEB 2011	
4. TITLE AND SUBTITLE  ACTIVE COOPERATION BETWEEN PRIMARY USERS AND COGNITIVE RADIO USERS IN HETEROGENEOUS AD-HOC NETWORKS				5a. CONTRACT NUMBER IN HOUSE	
				5b. GRANT NUMBER FA8750-11-1-0015	
				5c. PROGRAM ELEMENT NUMBER 62702F	
6. AUTHOR(S)  Weifeng Su, John Matyjas, and Stella Batalama				5d. PROJECT NUMBER ANCL	
				5e. TASK NUMBER 62	
				5f. WORK UNIT NUMBER 07	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) State University of New York at Buffalo Department of Electrical Engineering 332 Bonner Hall Buffalo, NY 14260-2050				8. PERFORMING ORGANIZATION REPORT NUMBER  N/A	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Air Force Research Laboratory/Information Directorate Rome Research Site/RITE 525 Brooks Road Rome NY 13441-4505				10. SPONSOR/MONITOR'S ACRONYM(S) AFRL/RI	
				11. SPONSORING/MONITORING AGENCY REPORT NUMBER AFRL-RI-RS-TP-2013-023	
12. DISTRIBUTION AVAILABILITY STATEMENT APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED. PA Case Number: 88ABW-2011-0329 DATE CLEARED: 9 JUNE 2010					
13. SUPPLEMENTARY NOTES © 2012 IEEE. IEEE Transactions on Signal Processing, Vol 60, No 4, April 2012. This work is copyrighted. One or more of the authors is a U.S. Government employee working within the scope of their Government job; therefore, the U.S. Government is joint owner of the work and has the right to copy, distribute, and use the work. All other rights are reserved by the copyright owner.					
14. ABSTRACT In this paper, we consider a heterogeneous ad-hoc network where primary users may cooperate with cognitive radio (CR) users for the transmission of their data. We propose a new cooperation protocol that allows CR users to relay primary user signals in exchange for some spectrum. The spectrum released by primary users is used by CR users for their own data transmission. The proposed protocol maximizes the primary user power savings and the CR users' own data transmission rate. In addition, it provides more robust (potentially continuous) service for CR users, compared to the conventional practice in cognitive networks where cognitive users transmit in the spectrum holes of primary users (i.e., their service is interrupted when primary users need to transmit and no spectrum holes are available). More specifically, we propose a CR user power allocation scheme that maximizes the rate of transmission of CR user own data, for any given CR user power budget and a given bandwidth released from the primary user. Furthermore, we determine a range of possible transmission power levels that can be used by the primary user during cooperation without sacrificing its target transmission rate, and we derive a necessary condition on the quality of the channel between the primary user and the CR user that enables cooperation. Extensive numerical and simulation studies illustrate our theoretical developments and show that cooperation between a primary and CR user may lead, for example, to up to 80% savings of primary user power when compared to a noncooperation scheme at the same transmission power level.					
15. SUBJECT TERMS Bandwidth , Cognitive radio , Data communication , Optimized production technology , Protocols , Relays , Resource management					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT  UU	18. NUMBER OF PAGES  11	19a. NAME OF RESPONSIBLE PERSON MICHAEL J. MEDLEY
a. REPORT U	b. ABSTRACT U	c. THIS PAGE U			19b. TELEPHONE NUMBER (Include area code) N/A

# Active Cooperation Between Primary Users and Cognitive Radio Users in Heterogeneous *Ad-Hoc* Networks

Weifeng Su, *Member, IEEE*, John D. Matyjias, *Member, IEEE*, and Stella Batalama, *Member, IEEE*

**Abstract**—In this paper, we consider a heterogeneous *ad-hoc* network where primary users may cooperate with cognitive radio (CR) users for the transmission of their data. We propose a new cooperation protocol that allows CR users to relay primary user signals in exchange for some spectrum. The spectrum released by primary users is used by CR users for their own data transmission. The proposed protocol maximizes the primary user power savings and the CR users' own data transmission rate. In addition, it provides more robust (potentially continuous) service for CR users, compared to the conventional practice in cognitive networks where cognitive users transmit in the spectrum holes of primary users (i.e., their service is interrupted when primary users need to transmit and no spectrum holes are available). More specifically, we propose a CR user power allocation scheme that maximizes the rate of transmission of CR user own data, for any given CR user power budget and a given bandwidth released from the primary user. Furthermore, we determine a range of possible transmission power levels that can be used by the primary user during cooperation without sacrificing its target transmission rate, and we derive a necessary condition on the quality of the channel between the primary user and the CR user that enables cooperation. Extensive numerical and simulation studies illustrate our theoretical developments and show that cooperation between a primary and CR user may lead, for example, to up to 80% savings of primary user power when compared to a noncooperation scheme at the same transmission power level.

**Index Terms**—Cooperative communications, amplify-and-forward relaying, cognitive radio, heterogeneous *ad-hoc* networks.

## I. INTRODUCTION

**C**OGNITIVE RADIO (CR) networks have attracted significant interest in recent years to pursue a natural idea that if spectrum is not used by primary users, secondary users may use it based on cognitive radio technologies [1]–[3]. Network spectrum efficiency can be greatly improved as secondary (or CR) users sense and exploit “spectrum holes” whenever they

are available. In the context of cognitive networks, the licensed users who own the spectrum (or users have higher priority to use the spectrum) are called primary users while nonlicensed users (or users with lower priority) are referred to as secondary users or CR users.

Cooperative communications is an emerging communication concept which optimizes signal transmissions both with respect to physical layer and medium-access-control (MAC) layer aspects (see [4]–[14] and the references therein). It has great potential to increase network capacity and power savings as well as reduce routing latency in wireless networks. Different from conventional point-to-point wireless communications, cooperative communications and networking allow different nodes/users in a wireless network to share resources to cooperatively deliver information to a destination through distributed transmissions. With this new communication concept, each node's information is sent out not only by the node itself, but also by cooperating nodes, and thus it is inherently more reliable for the destination to receive the information. Effectively, cooperating nodes create a virtual multiinput-multioutput (MIMO) system that can significantly increase the link capacity and realize a new form of spatial diversity which has been termed cooperative diversity.

Cooperative communication in cognitive heterogeneous networks can be achieved through either: (i) cooperation among primary user peers or (ii) cooperation among CR user peers or (iii) cooperation between primary and CR users. When only primary users are allowed to cooperate, case (i) above, cooperation reduces to the traditional cooperation scheme. When only CR users are allowed to cooperate, case (ii) above, cooperation is achieved through the means of a traditional cooperation scheme among CR users [15]–[17] enhanced by game theoretic principles to assist spectrum sharing among CR users [18], [19]. When cooperation is allowed between primary and CR users, case (iii) above, cooperation becomes challenging since primary users and CR users exhibit different priorities and may have security concerns for their own data. In [20], under the assumption that CR users know perfectly the data of primary users, it was shown that maximum rate can be achieved by simultaneous transmission of primary and CR user data over the same frequency, where CR user data are jointly encoded with primary user data via dirty-paper coding techniques [21]. In [22], a more realistic scheme was proposed where CR users utilize spectrum holes only whenever available and help forward primary user data packets that have not been successfully received by an intended destination. In this way, the need for data retransmission by primary users is eliminated which effectively results in power savings. The scheme was further generalized in [23] where, using dirty-paper coding techniques, CR users

Manuscript received July 12, 2011; revised October 26, 2011; accepted December 10, 2011. Date of publication January 02, 2012; date of current version March 06, 2012. The associate editor coordinating the review of this paper and approving it for publication is Prof. Yimin D. Zhang. This work was supported in part by the U.S. National Research Council (NRC) Research Fellowship Program and by the U.S. Air Force Research Laboratory (AFRL) by Grants FA87501110015 and FA87501110016. Approved for public release (reference number: 88ABW-2011-0329).

W. Su and S. Batalama are with the Department of Electrical Engineering, State University of New York (SUNY) at Buffalo, Buffalo, NY 14260 USA (e-mail: weifeng@buffalo.edu; batalama@buffalo.edu).

J. D. Matyjias is with the Air Force Research Laboratory/RIT, Rome, NY 13441 USA (e-mail: John.Matyjias@rl.af.mil).

Color versions of one or more of the figures in this paper are available online at <http://ieeexplore.ieee.org>.

Digital Object Identifier 10.1109/TSP.2011.2181841

could embed and transmit their own data while forwarding unsuccessful transmitted packets of primary user. In [24], a spectrum leasing scheme was proposed in which primary users may lease whole owned bandwidth for a fraction of time to an ad hoc network of secondary users based on decode-and-forward forwarding scheme and distributed space-time coding. In [25], it was proposed to deploy a “dumb” relay node in cognitive radio networks to help relay primary or CR user signals to improve network spectrum efficiency, where the protocol was analyzed and optimized for a network model consisting of a pair of primary users and a pair of CR users.

In this paper, we consider active cooperation between primary users and CR users in heterogeneous networks. In contrast with existing literature [20], we make a realistic assumption that primary user data are neither known nor perfectly decoded by CR users. We propose an active cooperation scheme between primary users and CR users which allows for more robust (potentially continuous) service/operation of the CR users in the network. This is, again, in contrast with existing work [22]–[25] where CR users transmit their own signals only if primary users are idle, resulting in CR data transmission that may be interrupted frequently. In the proposed protocol, primary users and CR users cooperate for mutual benefit—CR users assist to relay primary user signals in exchange for some spectrum released by the primary users. We understand that successful delivery of data at the lower possible power cost is, arguably, the most important objective of a primary user while the percentage of bandwidth used for data transmission is not critical. As a result, the primary user may release a portion of the bandwidth to the CR user for transmission of its own data. The amount of the released bandwidth may directly depend on the CR user effort needed to relay the primary user signals. Immediate benefits offered to the primary user by the above cooperation protocol include transmission power savings and on-air time reductions which can be critical, for example, in low-probability-interception (LPI) and low-probability-detection (LPD) military applications.

More specifically, in this paper we design a cognitive cooperation protocol that maximizes the primary user power savings and the CR user own data transmission rate. First, we develop an optimum power allocation scheme for the CR user that maximizes the CR user’s own data transmission rate for any given power budget and a given bandwidth released from the primary user. Then we determine a range of possible transmission power levels for the primary user to operate without sacrificing its own benefit in cooperation. We also find a necessary condition on the channel quality that allows cooperation between the primary user and the CR user. As it turns out, cooperation is triggered only if the amplitude of the channel between the primary user and the CR user is above a certain threshold. Numerical results show that the primary user enjoys significant average power savings as a result of the cooperation with CR users. It may save power up to 80% compared to the noncooperation case (the exact percentage of power savings depends on the channel condition).

The rest of this paper is organized as follows. In Section II, we specify a four-node cognitive radio network and describe the proposed cognitive cooperation protocol. In Section III, we optimize the CR user relaying power to maximize its own data transmission rate, based on its power budget. In Section IV, we discuss primary user options for cooperation, and determine a

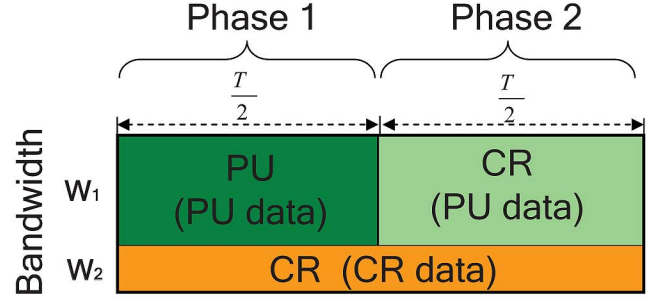


Fig. 1. Time-bandwidth allocation for PU and CR users.

minimum necessary power level as well as a maximum possible power level for the primary user to initiate cooperation without sacrificing its own transmission requirement. In Section V, we compare various strategies for the primary user power allocation in terms of primary user power savings and CR user own data transmission rate. Finally, some conclusions are drawn in Section VI.

## II. PROPOSED COGNITIVE COOPERATION PROTOCOL

For simplicity in presentation, we consider a basic four-node cognitive radio network consisting of a pair of primary users and a pair of CR users<sup>1</sup>. We assume that the bandwidth owned by the primary users is  $W$  Hz. For each primary user, we also assume that the target transmission rate is  $R_{PU}$  bits/s and the transmission power per unit frequency is  $P_1$  Watts/Hz. If CR users are available to assist with relaying primary user data, primary users may release some spectrum to CR users for their own use, as shown in Fig. 1. Specifically, let  $T$  denote the time duration that a primary user is allowed to transmit data over bandwidth  $W$  Hz. If a CR user is available to assist, then the primary user, without compromising its target transmission rate, may decide to transmit in only a part of the time slot (e.g.,  $\frac{T}{2}$ ) over a portion of the bandwidth,  $W_1 < W$  (Phase 1), hoping that cooperation with a CR user and data relaying for the rest of the time slot (Phase 2) may result in successful delivery of primary user data to the intended destination. The immediate benefits of cooperation for the primary user are on-air time reduction and transmission power savings. At the same time, the CR user assists to relay primary user signals in exchange for some bandwidth,  $W_2$  Hz, released by the primary user.

Let us assume that the primary user utilizes bandwidth  $W_1$  Hz and releases bandwidth  $W_2$  Hz to CR users ( $W_1 + W_2 = W$ ). Let  $x$  denote the signal transmitted by the primary user (source). The transmitted signal may be received by both the intended primary user (destination) and a nearby CR user (relay). The received signals at the destination ( $y_{s,d}$ ) and at the relay ( $y_{s,r}$ ) are given by

$$y_{s,d} = \sqrt{P_1} h_{s,d} x + \eta_{s,d} \quad (1)$$

$$y_{s,r} = \sqrt{P_1} h_{s,r} x + \eta_{s,r} \quad (2)$$

where  $h_{s,d}$  and  $h_{s,r}$  denote the channels between the primary source and the destination and between the source and the CR

<sup>1</sup>The proposed cognitive cooperation protocol and the theoretical development in this paper can be generalized to cognitive radio networks with more primary users and more CR users, in which each primary user may choose several CR users or the best CR user for cooperation.

relay, respectively. The terms  $\eta_{s,d}$  and  $\eta_{s,r}$  represent additive Gaussian noise and are modeled as zero-mean complex Gaussian random variables, with an average power per unit frequency  $\mathcal{N}_0 = 4.0 \times 10^{-21}$  W/Hz, (i.e.,  $-174$  dBm, which is the typical noise power density at room temperature) [26].

In this paper, we propose to utilize amplify-and-forward (AF), instead of decode-and-forward (DF), type cooperation/relaying where CR users simply amplify and forward primary user signals. Our choice is motivated by the fact that decoding/encoding at the relay may compromise the security and privacy of primary user data. Let us denote the CR relaying power as  $P_{2,1}$  W/Hz (power per unit frequency). Then the received signal at the destination is

$$y_{r,d} = \sqrt{P_{2,1}}\beta h_{r,d}y_{s,r} + \eta_{r,d} \quad (3)$$

where  $\beta = 1/\sqrt{P_1|h_{s,r}|^2 + \mathcal{N}_0}$  is a normalization factor that leads to received signal of unit average power. In (3),  $h_{r,d}$  denotes the channel coefficient between the CR user and the primary user's destination, and  $\eta_{r,d}$  is additive Gaussian noise with variance  $\mathcal{N}_0$ . Substituting (2) into (3), we obtain

$$y_{r,d} = \frac{\sqrt{P_1 P_{2,1}}}{\sqrt{P_1|h_{s,r}|^2 + \mathcal{N}_0}} h_{s,r} h_{r,d} x + \eta'_{r,d} \quad (4)$$

where

$$\eta'_{r,d} = \frac{\sqrt{P_{2,1}}}{\sqrt{P_1|h_{s,r}|^2 + \mathcal{N}_0}} h_{r,d} \eta_{s,r} + \eta_{r,d} \quad (5)$$

which is noise form with mean zero and variance  $(\frac{P_{2,1}|h_{r,d}|^2}{P_1|h_{s,r}|^2 + \mathcal{N}_0} + 1)\mathcal{N}_0$ . The channels  $h_{s,d}$ ,  $h_{s,r}$  and  $h_{r,d}$  are assumed to be independent complex Gaussian random variables with mean zero and variances  $\delta_{s,d}^2$ ,  $\delta_{s,r}^2$ , and  $\delta_{r,d}^2$ , respectively. The channel variances depend on the distance of the channel links as  $\delta_{i,j}^2 = (\frac{\lambda}{4\pi d_{i,j}})^\gamma$ , where  $d_{i,j}$  is the distance of a channel link,  $\lambda$  is the carrier wavelength (e.g., when the network operates at 1 GHz band,  $\lambda = 0.3$  m), and  $\gamma$  is the path loss coefficient. Without loss of generality we assume that the antenna gain is 1. For outdoor wireless networks, reasonable values for  $\gamma$  are  $\gamma = 2$  for channel links high above ground without multipath effects and  $\gamma = 3 \sim 4$  for channel links near ground. At the destination, the signal from the primary user in Phase 1 and the forwarded signal from the CR relay in Phase 2 are jointly decoded using maximum-ratio combining (MRC) [27].

The capacity of the resulting cooperative AF relay channel over bandwidth  $W_1$  Hz can be written as [6]

$$C_{\text{coop}} = \frac{W_1}{2} \times \log_2 \left[ 1 + \frac{P_1|h_{s,d}|^2}{\mathcal{N}_0} + f(P_1|h_{s,r}|^2, P_{2,1}|h_{r,d}|^2) \right] \quad (6)$$

where

$$f(P_1|h_{s,r}|^2, P_{2,1}|h_{r,d}|^2) = \frac{1}{\mathcal{N}_0} \frac{P_1 P_{2,1} |h_{s,r}|^2 |h_{r,d}|^2}{P_1|h_{s,r}|^2 + P_{2,1}|h_{r,d}|^2 + \mathcal{N}_0} \quad (7)$$

is the signal-to-noise ratio (SNR) at the destination resulted from the CR relaying. We note that the factor 1/2 in (6) is due

to the fact that the primary user utilizes only the first half of the time slot to transmit signals while the CR user utilizes the second half of the time slot to forward signals over the same bandwidth  $W_1$  Hz (Fig. 1). To meet the primary user target transmission rate,  $C_{\text{coop}}$  should be no less than  $R_{\text{PU}}$  bits/s, i.e.,

$$C_{\text{coop}} \geq R_{\text{PU}}. \quad (8)$$

The requirement in (8) implies that the bandwidth needed for the primary user should satisfy

$$W_1 \geq \frac{2R_{\text{PU}}}{\log_2 \left[ 1 + \frac{P_1|h_{s,d}|^2}{\mathcal{N}_0} + f(P_1|h_{s,r}|^2, P_{2,1}|h_{r,d}|^2) \right]}. \quad (9)$$

Then, the primary user may release the remaining bandwidth  $W - W_1$  Hz to CR users.

On the other hand, we understand that without any CR relaying, the primary user original transmission power  $P_0$  (Watts per unit frequency) must satisfy

$$W \log_2 \left( 1 + \frac{P_0|h_{s,d}|^2}{\mathcal{N}_0} \right) = R_{\text{PU}} \quad (10)$$

i.e.,

$$P_0 = \frac{\mathcal{N}_0}{|h_{s,d}|^2} \left( 2^{\frac{R_{\text{PU}}}{W}} - 1 \right). \quad (11)$$

Thus, in the absence of the relaying, the primary user data transmission takes place over the whole time slot  $T$ , and the primary user energy consumption amounts to  $P_0 W T$  joules. However, when CR relaying takes place (Fig. 1), the primary user transmits only in the first half of the time slot, so its energy consumption is only  $P_1 W_1 \frac{T}{2}$  joules. In this case, the rate of the primary user energy savings, defined as the ratio of the energy savings over the original energy consumption, is given by

$$\Phi \triangleq \frac{P_0 W T - P_1 W_1 \frac{T}{2}}{P_0 W T} = 1 - \frac{P_1 W_1}{2 P_0 W}. \quad (12)$$

Therefore, if the primary user maintains the transmission power level  $P_0$  (i.e.,  $P_1 = P_0$ ), then cooperation between the primary user and a CR user results in primary user energy savings of rate given by the following expression:

$$\Phi = 1 - \frac{W_1}{2W}. \quad (13)$$

From the above ratio, we can see that the less the bandwidth  $W_1$  that the primary user occupies [of course, it should satisfy the minimum bandwidth requirement in (9)], the more energy savings for the primary user. Note that in the calculation of primary user energy savings, we ignore the energy that the primary user spends on negotiating with the CR user in order to establish cooperation. A side benefit of using less bandwidth and shorter transmission time is the direct improvement of the LPD/LPI characteristics of the communication link that appears to be especially critical in military applications.

At the same time, the CR user can use the released bandwidth  $W_2 = W - W_1$  Hz for its own data transmission. Let us assume that the CR user has an energy budget equal to  $E_{\text{CR}}$  joules, and let us denote the power level used for CR user own

data transmission as  $P_{2,2}$  Watts/Hz (transmission power per unit frequency). Then,  $P_{2,2}$  should satisfy

$$P_{2,1}W_1\frac{T}{2} + P_{2,2}W_2T = E_{\text{CR}} \quad (14)$$

or equivalently

$$\frac{1}{2}P_{2,1}W_1 + P_{2,2}W_2 = P_{\text{CR}} \quad (15)$$

where  $P_{\text{CR}}$  is the average transmission power of the CR user in the time duration  $T$ . Thus, the CR user's potential own data transmission rate is

$$R_{\text{CR}} = W_2 \log_2 \left( 1 + \frac{P_{2,2}}{\mathcal{N}_0} |h_{\text{CR}}|^2 \right) \quad (16)$$

where  $h_{\text{CR}}$  is the channel between the CR user and its own destination. We immediately observe a tradeoff. The more CR power is allocated to the relaying of primary user signals, the more bandwidth gets released to the CR user who, now, has less remaining power for its own data transmission. So, the CR user has to determine how much power should be allocated to the relaying process to maximize its own data transmission rate.

### III. CR USER: RELAYING POWER OPTIMIZATION

In this section, given a CR user power budget, we propose a power allocation scheme that determines the CR power level that is used for relaying primary user signals and the power level that is used for CR user own data transmission. More specifically, for any given CR power budget  $P_{\text{CR}}$ , let  $\alpha$  denote the ratio of the CR power allocated to assist with relaying primary user signals over the CR power budget, i.e.

$$\frac{1}{2}P_{2,1}W_1 = \alpha P_{\text{CR}}, \text{ and } P_{2,2}W_2 = (1 - \alpha)P_{\text{CR}} \quad (17)$$

where  $\alpha \in [0, 1]$ . We note that, if  $\alpha$  is too small, the allocated relaying power may not be enough to trigger cooperation and consequently no bandwidth is released from the primary user. On the other hand, if  $\alpha$  is large, the remaining power may be insufficient for CR user's own data transmission. Motivated by the above observation, we propose a power allocation scheme that selects the power ratio  $\alpha$  to maximize the CR user own data transmission rate, subject to any given CR user power budget and a range of values of potential bandwidth released from the primary user.

For any given ratio  $\alpha \in [0, 1]$ , based on (9) and the fact that  $\frac{1}{2}P_{2,1}W_1 = \alpha P_{\text{CR}}$ , the corresponding relaying power  $P_{2,1}$  can be determined by the following:

$$\frac{P_{2,1}R_{\text{PU}}}{\log_2 \left[ 1 + \frac{P_1|h_{s,d}|^2}{\mathcal{N}_0} + f(P_1|h_{s,r}|^2, P_{2,1}|h_{r,d}|^2) \right]} = \alpha P_{\text{CR}} \quad (18)$$

where  $f(P_1|h_{s,r}|^2, P_{2,1}|h_{r,d}|^2)$  is specified in (7). If we define

$$G(x) \triangleq \ln \left[ 1 + \frac{P_1|h_{s,d}|^2}{\mathcal{N}_0} + f(P_1|h_{s,r}|^2, x|h_{r,d}|^2) \right] - x \frac{R_{\text{PU}} \ln 2}{\alpha P_{\text{CR}}} \quad (19)$$

then the proper relaying power  $P_{2,1}$  can be determined by solving the equation  $G(P_{2,1}) = 0$ . For simplicity of presentation, let us denote

$$a = 1 + \frac{P_1|h_{s,d}|^2}{\mathcal{N}_0}, \quad b_1 = \frac{P_1|h_{s,r}|^2}{\mathcal{N}_0}, \\ b_2 = \frac{P_1|h_{s,r}|^2 + \mathcal{N}_0}{|h_{r,d}|^2}, \quad c = \frac{R_{\text{PU}} \ln 2}{\alpha P_{\text{CR}}} \quad (20)$$

then the function in (19) can be written as

$$G(x) = \ln \left( a + \frac{b_1 x}{b_2 + x} \right) - cx. \quad (21)$$

The following Lemma identifies two properties of the function  $G(x)$  that play a key role in solving  $G(P_{2,1}) = 0$  and thus determining proper relaying power level  $P_{2,1}$  (the proof is included in Appendix I).

*Lemma 1:* For any given ratio  $\alpha \in [0, 1]$  of the CR power allocated to the relaying of primary user signals over the total CR power budget, let

$$x^* = \frac{-(2a + b_1)b_2 + \sqrt{b_1^2 b_2^2 + 4(a + b_1)\frac{b_1 b_2}{c}}}{2(a + b_1)} \quad (22)$$

where  $a, b_1, b_2$  and  $c$  are specified in (20). Then the function  $G(x)$  in (21) exhibits the following properties:

- i) If  $c < \frac{b_1}{ab_2}$ , the function  $G(x)$  is increasing for  $0 \leq x \leq x^*$ , and decreasing for  $x > x^*$ ;
- ii) If  $c \geq \frac{b_1}{ab_2}$ , the function  $G(x)$  is decreasing for  $x \geq 0$ .

Furthermore, the equation  $G(x) = 0$  has a unique positive solution, denoted as  $x_0$ , which is located in the following range:

$$\max \left( x^*, \frac{\ln a}{c} \right) < x_0 < \frac{\ln(a + b_1)}{c}. \quad (23)$$

From Lemma 1, we can see that for any given ratio  $\alpha \in [0, 1]$ , the equation  $G(P_{2,1}) = 0$  has a unique positive solution within the interval  $[\max(x^*, \frac{\ln a}{c}), \frac{\ln(a+b_1)}{c}]$ . Since the function  $G(x)$  is decreasing over the interval  $[\max(x^*, \frac{\ln a}{c}), \frac{\ln(a+b_1)}{c}]$ , the solution of  $G(x) = 0$  can be easily determined by the Newton method. Therefore, we can determine a unique relaying power  $P_{2,1}$  from the equation  $G(P_{2,1}) = 0$ , where the function  $G(x)$  is defined in (19), for any given power ratio  $\alpha \in [0, 1]$ .

With the proper relaying power  $P_{2,1}$  determined by solving the equation  $G(P_{2,1}) = 0$  for any given ratio  $\alpha \in [0, 1]$ , according to (9), a corresponding minimum bandwidth  $W_1$  can be determined as

$$W_1 = \frac{2R_{\text{PU}}}{\log_2 \left[ 1 + \frac{P_1|h_{s,d}|^2}{\mathcal{N}_0} + f(P_1|h_{s,r}|^2, P_{2,1}|h_{r,d}|^2) \right]}. \quad (24)$$

If  $W_1 > W$ , then there will be no cooperation since the primary user target data rate cannot be achieved. So, the CR user has to choose a power ratio  $\alpha$  such that the corresponding bandwidth  $W_1$  is not larger than the total bandwidth  $W$ . Once  $W_1 (W_1 \leq W)$  is determined, the bandwidth that can be released to the CR user is  $W_2 = W - W_1$ . Then, (17) implies that the CR user

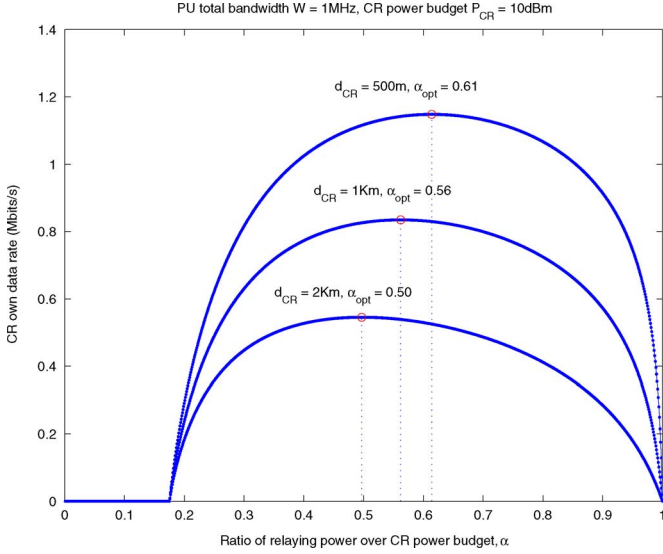


Fig. 2. CR user (own) data rate as a function of the ratio of CR user relaying power over CR total power budget.  $d_{sd} = d_{rd} = 1$  Km and  $d_{sr} = 500$  m.

power  $P_{2,2}$  that is used for its own data transmission should satisfy the following:

$$P_{2,2} = \frac{(1 - \alpha)P_{CR}}{W_2}. \quad (25)$$

Substituting (25) into (16), the CR user's own data transmission rate is then given by

$$R_{CR} = (W - W_1) \log_2 \left[ 1 + \frac{(1 - \alpha)P_{CR}}{(W - W_1)N_0} |h_{CR}|^2 \right] \quad (26)$$

which should be maximized by selecting proper power ratio  $\alpha$ .

In Fig. 2, we plot the CR user rate for the transmission of its own data as a function of power ratio  $\alpha$  ( $0 \leq \alpha \leq 1$ ). We assume that the primary user has total bandwidth  $W = 1$  MHz and target data rate  $R_{PU} = 1$  Mbits/s, while the CR user has a power budget  $P_{CR} = 10$  dBm. The network operates at 1 GHz band and the path loss coefficient is  $\gamma = 2$ . In this example, we consider the scenario where the primary user maintains the same transmission power level (i.e.,  $P_1 = P_0$ ) based on (11). We assume that the distance between the primary user and its destination is 1 Km, the distance between the primary user and the relaying CR user is 500 m, and the distance between the CR user and the destination is 1 Km. Moreover, we assume that the channels suffer path loss which depend on the distance of the channel links as  $(\frac{\lambda}{4\pi d_{i,j}})^2$ . We plot the CR user own data rate for three different values of the distance between the CR user and its own destination, namely 500 m, 1 Km and 2 Km. From the figure, we can observe that the optimum ratio of the relaying power over the total power budget is 0.61, 0.56, and 0.50 for the cases of  $d_{CR} = 500$  m, 1 Km, and 2 Km, respectively. For this system, the corresponding energy saving for the primary user due to cooperation is 58%, 57.8%, and 57.3%, respectively.

In Fig. 3, we repeat the same study as in Fig. 2, with the only difference that the distance between the primary user and the relaying CR user is 100 m. We observe that the optimum ratio of

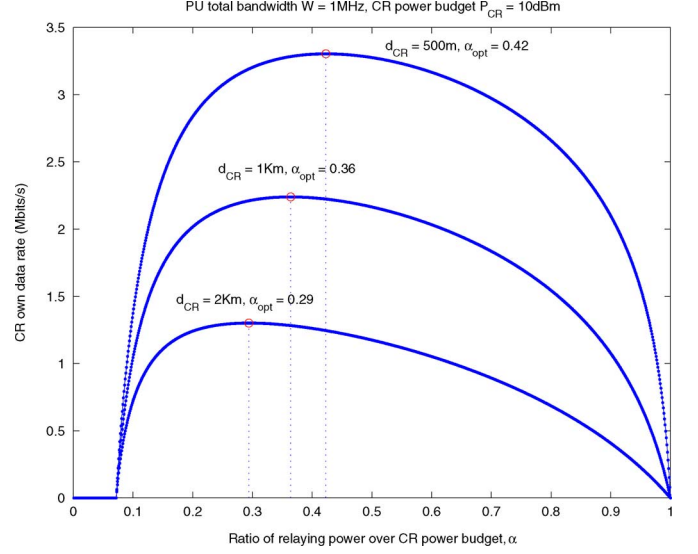


Fig. 3. CR user (own) data rate as a function of the ratio of CR user relaying power over CR total power budget.  $d_{sd} = d_{rd} = 1$  Km and  $d_{sr} = 100$  m.

the relaying power over the total power budget is 0.42, 0.36, and 0.29 for  $d_{CR} = 500$  m, 1 Km, and 2 Km, respectively. For this system setup, the corresponding energy savings for the primary user due to cooperation is 78%, 77%, and 75%, respectively. Direct comparison of Figs. 2 and 3 show that the smaller the distance between the relaying CR user and the primary user, the more the primary user power savings due to cooperation, and the higher the CR user own data rate.

#### IV. PRIMARY USER: COOPERATION OPTIONS

The primary user has priority to decide whether or not to cooperate with the CR user. As the spectrum owner, the primary user is the one who decides how much spectrum to release and what level of its own transmission power to use. Let the transmission power that the primary user decides to use be equal to  $P_1 = \zeta P_0$  Watts/Hz, where  $P_0 = \frac{N_0}{|h_{s,d}|^2} (2^{\frac{R_{PU}}{W}} - 1)$  Watts/Hz is its original transmission power (before a decision to cooperate is made), and  $\zeta$  is a parameter to be determined. In the following, we determine the range of the parameter  $\zeta$  and examine its impact on the primary user energy savings, as well as the CR user own potential data rate.

First, we determine the minimum value of the parameter  $\zeta$ , or equivalently the minimum necessary transmission power of the primary user, below which its target data rate cannot be guaranteed regardless of the amount of the CR user effort to relay primary user signals. Since the total bandwidth is  $W$  Hz, based on (9), we have

$$W \geq W_1 + \frac{2R_{PU}}{\log_2 \left[ 1 + \frac{\zeta P_0 |h_{s,d}|^2}{N_0} + f(\zeta P_0 |h_{s,r}|^2, P_{2,1} |h_{r,d}|^2) \right]}. \quad (27)$$

Thus, we have a constraint on  $\zeta$  as follows

$$\frac{\zeta P_0 |h_{s,d}|^2}{N_0} + f(\zeta P_0 |h_{s,r}|^2, P_{2,1} |h_{r,d}|^2) \geq 2^{\frac{2R_{PU}}{W}} - 1. \quad (28)$$



From (7), we observe that (due to the fact that  $\frac{AB}{A+B} \leq \min\{A, B\}$  for any positive  $A$  and  $B$ )

$$f(\zeta P_0 |h_{s,r}|^2, P_{2,1} |h_{r,d}|^2) < \min \left\{ \frac{\zeta P_0 |h_{s,r}|^2}{\mathcal{N}_0}, \frac{P_{2,1} |h_{r,d}|^2}{\mathcal{N}_0} \right\} \quad (29)$$

and  $f(\zeta P_0 |h_{s,r}|^2, P_{2,1} |h_{r,d}|^2)$  is an increasing function in terms of the power  $P_{2,1}$ . Note that the upper bound in (29) is tight for moderate or high power level. Moreover, when the CR relaying power  $P_{2,1}$  is high enough,  $f(\zeta P_0 |h_{s,r}|^2, P_{2,1} |h_{r,d}|^2)$  converges to  $\frac{\zeta P_0 |h_{s,r}|^2}{\mathcal{N}_0}$ . From the above observation and (28), we have

$$\frac{\zeta P_0 (|h_{s,d}|^2 + |h_{s,r}|^2)}{\mathcal{N}_0} \geq 2^{\frac{2R_{PU}}{W}} - 1. \quad (30)$$

Therefore, the parameter  $\zeta$  is lower bounded as

$$\zeta \geq \zeta_{\min} \triangleq \frac{|h_{s,d}|^2}{|h_{s,d}|^2 + |h_{s,r}|^2} \left( 2^{\frac{2R_{PU}}{W}} + 1 \right) \quad (31)$$

which is the minimum value of the parameter that the primary user may consider, i.e., the primary user transmission power should not be less than  $\zeta_{\min} P_0$  Watts/Hz.

Next, we evaluate the maximum value of the parameter  $\zeta$  that the primary user may choose to determine its transmission power. Beyond that value, the primary user cannot save any energy from cooperation, and for this reason there is no incentive to cooperate with the CR user. Since the average transmission power of the primary user over the bandwidth  $W_1$  is  $\frac{1}{2} P_1 W_1$  (which should not be larger than  $P_0 W$ ), it is implied that

$$\frac{\zeta P_0 R_{PU}}{\log_2 \left[ 1 + \frac{\zeta P_0 |h_{s,d}|^2}{\mathcal{N}_0} + f(\zeta P_0 |h_{s,r}|^2, P_{2,1} |h_{r,d}|^2) \right]} \leq P_0 W. \quad (32)$$

Since  $f(\zeta P_0 |h_{s,r}|^2, P_{2,1} |h_{r,d}|^2)$  is upper bounded by  $\frac{\zeta P_0 |h_{s,r}|^2}{\mathcal{N}_0}$ , we have

$$\frac{\zeta}{\log_2 \left[ 1 + \frac{\zeta P_0}{\mathcal{N}_0} (|h_{s,d}|^2 + |h_{s,r}|^2) \right]} \leq \frac{W}{R_{PU}}. \quad (33)$$

Substituting  $P_0 = \frac{\mathcal{N}_0}{|h_{s,d}|^2} (2^{\frac{2R_{PU}}{W}} - 1)$  into the above inequality, we obtain another constraint for the parameter  $\zeta$

$$\frac{\zeta}{\log_2 \left[ 1 + \zeta \left( 1 + \frac{|h_{s,r}|^2}{|h_{s,d}|^2} \right) \left( 2^{\frac{2R_{PU}}{W}} - 1 \right) \right]} \leq \frac{W}{R_{PU}}. \quad (34)$$

Let us denote the left-hand side (LHS) of the above inequality as

$$F(\zeta) \triangleq \frac{\zeta}{\log_2 [1 + \zeta A]} \quad (35)$$

where

$$A = \left( 1 + \frac{|h_{s,r}|^2}{|h_{s,d}|^2} \right) \left( 2^{\frac{2R_{PU}}{W}} - 1 \right). \quad (36)$$

The function  $F(\zeta)$  has the following property (the proof is included in Appendix II).

*Lemma 2:* The function  $F(\zeta)$  in (35) is increasing for any  $\zeta \geq 0$ . Moreover,  $F(\zeta_{\min}) \leq \frac{W}{R_{PU}}$  if and only if

$$|h_{s,r}|^2 \geq \frac{2^{\frac{2R_{PU}}{W}} - 1}{2} |h_{s,d}|^2 \quad (37)$$

which is a necessary condition for cooperation.

From Lemma 2, we can see that the LHS of the constraint for the parameter  $\zeta$  in (34) is an increasing function of  $\zeta$ , and it goes to infinity when  $\zeta$  goes to infinity. Thus, if  $F(\zeta_{\min}) \leq \frac{W}{R_{PU}}$ , then there exists a unique solution, denoted as  $\zeta_{\max}$ , satisfying the constraint in (34) with equality, and the solution can be easily obtained by using the Newton method. Then, the maximum transmission power that the primary user can use without sacrificing its own benefit in cooperation is  $\zeta_{\max} P_0$  Watts/Hz. On the other hand, if  $F(\zeta_{\min}) > \frac{W}{R_{PU}}$ , it implies that there is no parameter  $\zeta \geq \zeta_{\min}$  satisfying the constraint (34), and thus the primary user cannot gain any power savings from cooperation. Therefore, a necessary condition for cooperation is  $F(\zeta_{\min}) \leq \frac{W}{R_{PU}}$

which is equivalent to  $|h_{s,r}|^2 \geq \frac{2^{\frac{2R_{PU}}{W}} - 1}{2} |h_{s,d}|^2$ . In other words, the channel between the primary user and the CR user,  $h_{s,r}$ , should be reasonably good to enable cooperation. If the channel  $h_{s,r}$  is bad, the primary user cannot benefit any power savings to cooperate with the CR user.

## V. PRIMARY USER: TRANSMISSION POWER ALLOCATION STRATEGIES

As shown in the previous section, the primary user may choose any value of the parameter  $\zeta$  ( $\zeta_{\min} \leq \zeta \leq \zeta_{\max}$ ) to determine its transmission power  $\zeta P_0$  Watts/Hz in cooperation. In this section, we compare different strategies for the selection of the primary user transmission power in terms of resulting primary user power savings and corresponding maximum transmission rate for CR user data. In all simulation studies, we assume that the network operates at the 10 GHz frequency band and the primary user has total bandwidth  $W = 1$  MHz and a target data rate  $R_{PU} = 1$  Mbits/s.

In Figs. 4 and 5, we plot the primary user power savings as well as the corresponding maximum transmission rate for CR user own data by varying the power parameter  $\zeta$  ( $\zeta_{\min} \leq \zeta \leq \zeta_{\max}$ ). We assume that all the channels have distance of 1 Km, except for the distance between the primary user and the CR user which is  $d_{s,r} = 500$  m and 100 m in Figs. 4 and 5, respectively. Moreover, we assume that the channels suffer only path loss which depend on the distance of the channel links as  $(\frac{\lambda}{4\pi d_{i,j}})^\gamma$  and  $\gamma = 2$ . In each figure, we consider two different values of CR user power budget, namely  $P_{CR} = 20$  dBm and  $P_{CR} = 10$  dBm. For any given primary user power parameter  $\zeta$ , the primary user power savings is calculated based on the optimized CR relaying power as determined in Section III. We observe that the primary user power savings can be up to 70% in Fig. 4 where  $d_{s,r} = 500$  m and the power savings can be up to 90% in Fig. 5 where  $d_{s,r} = 100$  m. The smaller the value of the power parameter  $\zeta$ , the more the power savings of the primary user and the less the benefits for the CR user. On the other hand, the larger the value of the parameter  $\zeta$ , the less the power savings of the primary user and

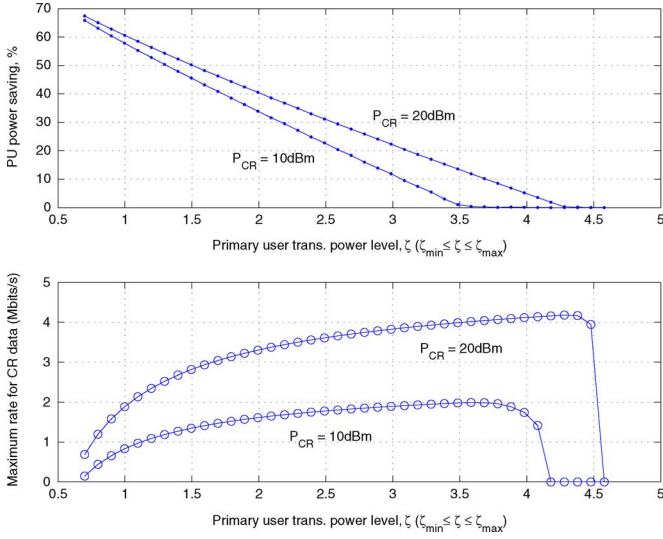


Fig. 4. Primary user power savings and corresponding maximum rate for CR user own data,  $d_{sr} = 500$  m.

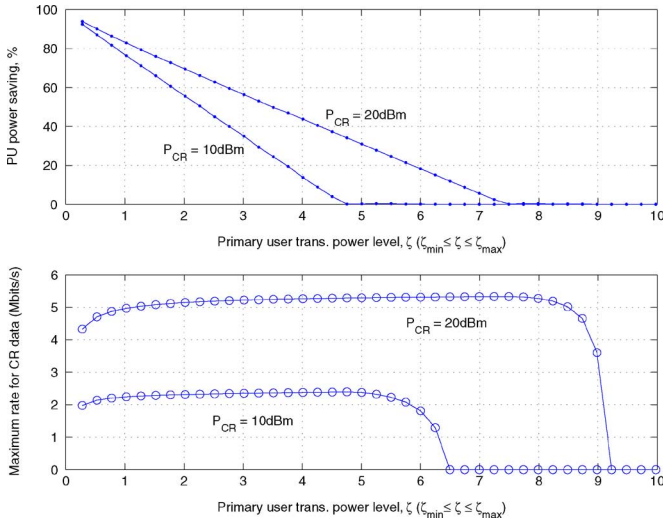


Fig. 5. Primary user power savings and corresponding maximum rate for CR user own data,  $d_{sr} = 100$  m.

the more the benefits for the CR user. In Figs. 4 and 5, we also plot the maximum transmission rate of the CR user own data as a function of the power parameter  $\zeta$  for two different values of CR user power budget, i.e.,  $P_{CR} = 20$  dBm and  $P_{CR} = 10$  dBm. We can see that the CR user own data rate increases when the primary user's transmission power level is increased, but the increasing of the CR user data rate is not linear.

As the spectrum owner, the primary user has the priority to set its transmission power level for cooperation by choosing the power parameter  $\zeta$  ( $\zeta_{\min} \leq \zeta \leq \zeta_{\max}$ ). It can be either aggressive, moderate, or generous. We compare the following schemes:

- Scheme A:  $P_1 = \zeta P_0$  with  $\zeta = \zeta_{\min}$
- Scheme B:  $P_1 = \zeta P_0$  with  $\zeta = \zeta_{\max}$
- Scheme C:  $P_1 = \zeta P_0$  with  $\zeta = 1$
- Scheme D:  $P_1 = \zeta P_0$  with  $\zeta = 2$
- Scheme E:  $P_1 = \zeta P_0$  with  $\zeta = \max(\zeta_{\min}, 1)$

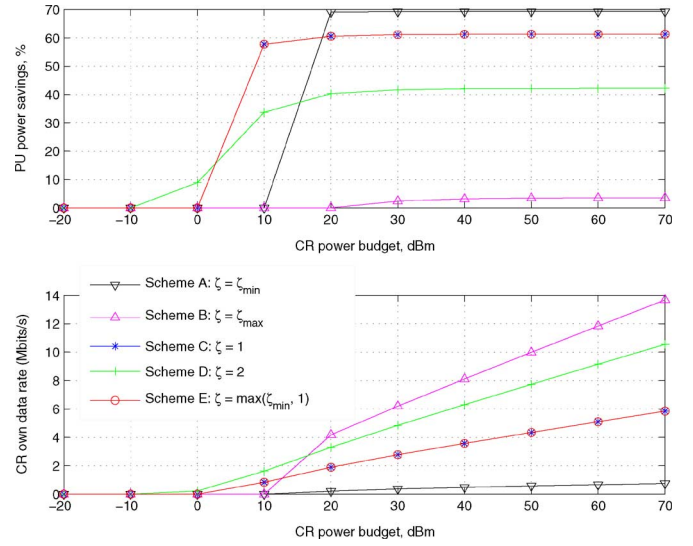


Fig. 6. Comparison of different strategies for the primary user to choose its transmission power level in *nonfading* environment,  $d_{sr} = 500$  m.

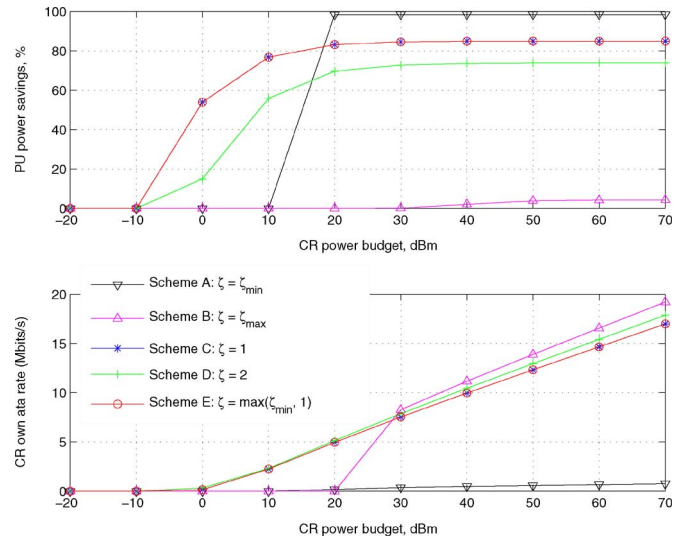


Fig. 7. Comparison of different strategies for the primary user to choose its transmission power level in *nonfading* environment,  $d_{sr} = 100$  m.

In Scheme A, the primary user chooses the minimum necessary transmission power level, so it gains the most in power savings and the benefit for the CR user is minimum, which is the most aggressive strategy for the primary user. In Scheme B, on the other hand, the primary user chooses the maximum transmission power level, so it gains the least in power savings and the benefit for the CR user is maximum—that is the most generous strategy on behalf of primary user. Figs. 4 and 5 show a power parameter  $\zeta = 1$ –2 can be a good choice for the primary user as it captures most of the power savings and the corresponding CR data rate reaches almost its maximum—this is the rationale for Schemes C and D. When  $\zeta = 1$ , it means the primary user considers the same transmission power level as in a noncooperation situation. Scheme E is a combination of Scheme A and Scheme C which can be a good tradeoff for the primary user power savings and the CR user own data rate as shown later.



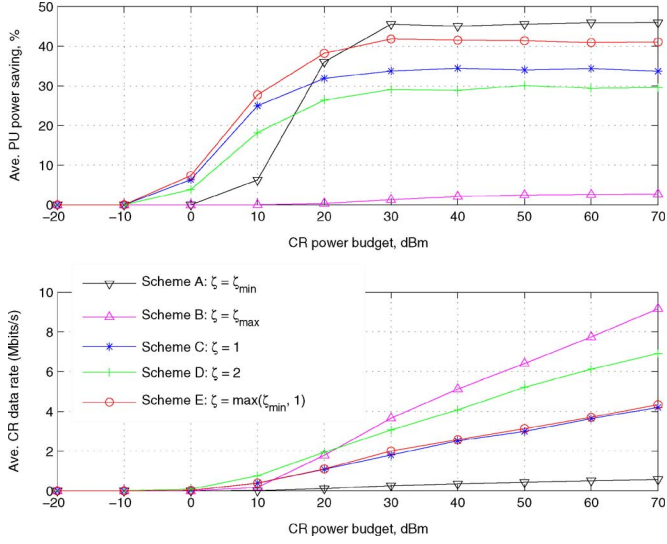


Fig. 8. Comparison of different strategies for the primary user to choose its transmission power level in fading environment,  $d_{sr} = 500$  m.

In Figs. 6 and 7, we compare the performance of Schemes A-E in terms of primary user power savings and the corresponding CR user data rate in nonfading environment. We assume that all the channels have distance of 1 Km, except for the distance between the primary user and the CR user which is  $d_{sr} = 500$  m in Fig. 6 and  $d_{sr} = 100$  m in Fig. 7, respectively. Moreover, we assume that the channels depend on the distance of the channel links as  $(\frac{\lambda}{4\pi d_{i,j}})^2$ . The primary user power savings and the corresponding CR user data rate are shown with different CR power budget. We can see from both figures that Scheme A leads to the best performance to the primary user power savings but the worst to the CR user data rate. Scheme B gives the best performance to the CR user data rate but the worst to the primary user power savings. The performance of these two schemes provides benchmark or guideline for the primary user. The performances of other schemes fall into the range of the two benchmark schemes. The performance of Scheme C and Scheme D are moderate. It is interesting to observe that with Scheme C, i.e., the primary user maintains the same transmission power as that in a noncooperation case, the primary user can save power up to 42% and 85% in Figs. 6 and 7, respectively. Moreover, the performance of Scheme E is the same as that of Scheme C in this nonfading scenario.

In Figs. 8 and 9, we compare the performance of Schemes A-E in terms of the average primary user power savings and the corresponding average CR user data rate in a Rayleigh fading environment. We assume that all channel links are independent Rayleigh fading links with variances depending on link distance as  $(\frac{\lambda}{4\pi d_{i,j}})^2$ . All the channels have distance of 1 Km, except for the distance between the primary user and the CR user which is  $d_{sr} = 500$  m in Fig. 8 and  $d_{sr} = 100$  m in Fig. 9. The average primary user power savings and the corresponding average CR user data rate are shown in terms of different CR power budget. From both figures, we can see that Scheme A gives the best performance to the average primary user power savings but the worst to the average CR user data rate. Scheme B gives the best performance to the average CR user data rate but the worst to the average primary user power savings. The performances of

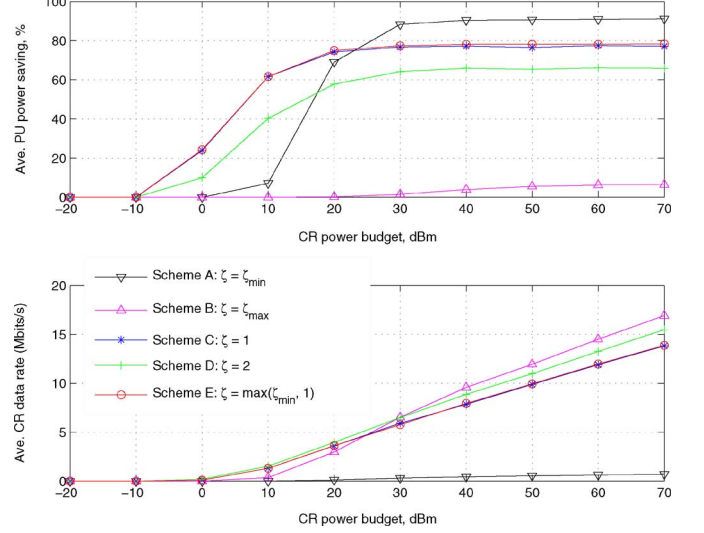


Fig. 9. Comparison of different strategies for the primary user to choose its transmission power level in fading environment,  $d_{sr} = 100$  m.

other schemes fall into the range of the two benchmark Schemes A and B. It is interesting to observe that with Scheme C, i.e., the primary user maintains the same transmission power as that in the noncooperation case, the primary can save average power up to 33% and 78% in Figs. 8 and 9, respectively. Scheme E takes advantage of both Schemes A and C. It shows that in terms of the average CR user data rate, Scheme E performs almost the same as Scheme C. However, in terms of the average primary user power savings, the performance of Scheme E is much better than that of Scheme A. Therefore, Scheme E is a good choice for the primary user in fading scenario.

## VI. CONCLUSION

In this paper, we proposed a cognitive cooperation protocol for cognitive heterogeneous *ad-hoc* networks, in which primary and CR users may actively cooperate for mutual benefits. CR users may assist to relay primary user signals in exchange for some spectrum released by the primary users. We determined an optimum power allocation scheme for the CR user to allocate relaying power level. For any given CR user power budget and a given bandwidth released from the primary user, our algorithm maximizes the CR user own data transmission rate. We also determined a necessary condition on the quality of the channel between the primary user and the CR user for them to cooperate. It turns out that to enable cooperation, the amplitude of the channel between the primary user and the CR user has to be above a certain threshold. Immediate primary user benefits due to cooperation are transmission power savings and on-air time reduction. We note that the primary user has the priority to choose its transmission power level during cooperation while the amount of bandwidth released is based on the order of its power savings. We have designed a methodology that evaluates a minimum necessary power level and a maximum possible power level that the primary user may choose during cooperation without sacrificing its own target rate. Extensive numerical and simulation studies illustrated our theoretical developments and showed that, when compared to a noncooperation scheme at the same transmission power level, the primary users may enjoy significant power savings (for example up to 33% and 78%) due

to cooperation. Note that in this paper, to better present and understand the basic idea, we ignore the overhead energy that primary users and CR users spend on interacting to establish cooperation. However, our theoretical development and optimization can be easily extended to include the overhead energy spent on establishing cooperation.

#### APPENDIX I PROOF OF LEMMA 1

The derivative of the function  $G(x)$  in (21) is

$$\frac{\partial G(x)}{\partial x} = \frac{1}{a + \frac{b_1 x}{b_2 + x}} \cdot \frac{b_1 b_2}{(b_2 + x)^2} - c. \quad (38)$$

We can see that  $\frac{\partial G(x)}{\partial x} > 0$  is equivalent to

$$\left(a + \frac{b_1 x}{b_2 + x}\right) (b_2 + x)^2 < \frac{b_1 b_2}{c} \quad (39)$$

i.e.,

$$(a + b_1)x^2 + (2a + b_1)b_2 x + ab_2^2 - \frac{b_1 b_2}{c} < 0. \quad (40)$$

The inequality in (40) can be solved as

$$\frac{-(2a + b_1)b_2 - \sqrt{\Delta}}{2(a + b_1)} < x < \frac{-(2a + b_1)b_2 + \sqrt{\Delta}}{2(a + b_1)} \quad (41)$$

where

$$\begin{aligned} \Delta &= (2a + b_1)^2 b_2^2 - 4(a + b_1) \left(ab_2^2 - \frac{b_1 b_2}{c}\right) \\ &= b_1^2 b_2^2 + 4(a + b_1) \frac{b_1 b_2}{c}. \end{aligned} \quad (42)$$

With the notation

$$x^* = \frac{-(2a + b_1)b_2 + \sqrt{b_1^2 b_2^2 + 4(a + b_1) \frac{b_1 b_2}{c}}}{2(a + b_1)}$$

we can see that  $x^* > 0$  is valid if and only if  $c < \frac{b_1}{ab_2}$ . Therefore, when  $c < \frac{b_1}{ab_2}$ , according to (39)–(41), we conclude that  $\frac{\partial G(x)}{\partial x} > 0$  for any  $x$  within the interval  $0 \leq x \leq x^*$ , i.e., the function  $G(x)$  is increasing within the interval  $0 \leq x \leq x^*$ . Furthermore, when  $c < \frac{b_1}{ab_2}$ , based on (39)–(41) we can also see that  $\frac{\partial G(x)}{\partial x} < 0$  for any  $x > x^*$ , i.e., the function  $G(x)$  is decreasing for any  $x > x^*$ . On the other hand, when  $c \geq \frac{b_1}{ab_2}$  which implies  $x^* \leq 0$ , from (38)–(41) we can see that  $\frac{\partial G(x)}{\partial x} < 0$  is true for any  $x > 0$ , i.e., the function  $G(x)$  is decreasing for  $x \geq 0$  in this case.

From the above discussion, we know that the function  $G(x)$  is decreasing for any  $x \geq \max(x^*, 0)$ . Since  $G(0) = \ln a > 0$  ( $a > 1$ ) and  $G(x) \rightarrow -\infty$  when  $x$  goes to  $\infty$ , we conclude that the equation  $G(x) = 0$  has a unique positive solution. Let us denote the positive solution as  $x_0$ , then according to (21), it must satisfy

$$x_0 = \frac{\ln \left(a + \frac{b_1 x_0}{b_2 + x_0}\right)}{c}. \quad (43)$$

Since  $0 < \frac{b_1 x_0}{b_2 + x_0} < b_1$ , so

$$\frac{\ln a}{c} < x_0 < \frac{\ln(a + b_1)}{c}. \quad (44)$$

Therefore, we prove the Lemma completely.  $\square$

#### APPENDIX II PROOF OF LEMMA 2

First, we would like to show that the function  $F(\zeta)$  in (35) is increasing for any  $\zeta \geq 0$ . The derivative of the function  $F(\zeta)$  is

$$\frac{\partial F(\zeta)}{\partial \zeta} = \frac{\ln(1 + \zeta A) - \frac{\zeta A}{1 + \zeta A}}{\ln^2(1 + \zeta A)}. \quad (45)$$

Thus, it is sufficient to show that the numerator in (45) is positive for any  $\zeta \geq 0$ . Let us further denote  $\tilde{F}(\zeta) \triangleq \ln(1 + \zeta A) - \frac{\zeta A}{1 + \zeta A}$ , and we have

$$\frac{\partial \tilde{F}(\zeta)}{\partial \zeta} = \frac{A}{1 + \zeta A} - \frac{A}{(1 + \zeta A)^2}. \quad (46)$$

Since  $A$  is positive, we observe that  $\frac{\partial \tilde{F}(\zeta)}{\partial \zeta} > 0$  for any  $\zeta \geq 0$ , which implies that  $\tilde{F}(\zeta)$  is increasing for any  $\zeta \geq 0$ . Since  $\tilde{F}(0) = 0$ , so we can conclude that  $\tilde{F}(\zeta)$  is positive for any  $\zeta \geq 0$ . Therefore, the function  $F(\zeta)$  is increasing for any  $\zeta \geq 0$ .

Next, we determine the necessary condition for the primary user and the CR user to cooperate. For any possible parameter  $\zeta$ , it should satisfy the constraint (34), otherwise the primary user cannot gain any power savings from the cooperation. So, the minimum parameter  $\zeta_{\min}$  in (31) should also satisfy the constraint (34), which leads to a necessary condition for cooperation as

$$F(\zeta_{\min}) \leq \frac{W}{R_{\text{PU}}}. \quad (47)$$

i.e.,

$$\frac{\zeta_{\min}}{\log_2 \left[ 1 + \zeta_{\min} \left( 1 + \frac{|h_{s,r}|^2}{|h_{s,d}|^2} \right) \left( 2^{\frac{R_{\text{PU}}}{W}} - 1 \right) \right]} \leq \frac{W}{R_{\text{PU}}}. \quad (48)$$

By substituting the minimum parameter  $\zeta_{\min}$  (31) into the above inequality, it is not difficult to show that the inequality in (48) is equivalent to

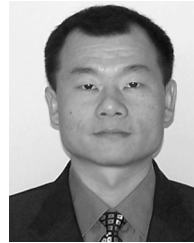
$$|h_{s,r}|^2 \geq \frac{2^{\frac{R_{\text{PU}}}{W}} - 1}{2} |h_{s,d}|^2. \quad (49)$$

Therefore, we prove Lemma 2 completely.  $\square$

#### REFERENCES

- [1] Federal Communications Commission, Notice of Proposed Rule Making and Order ET Docket No. 03-322, Dec. 2003.
- [2] J. Mitola, III and G. Q. Maguire, Jr., "Cognitive radio: Making software radio more personal," *IEEE Pers. Commun.*, pp. 13–18, Aug. 1999.
- [3] F. K. Jondral, "Cognitive radio: A communications engineering view," *IEEE Wireless Commun.*, pp. 28–33, Aug. 2007.
- [4] E. C. van der Meulen, "A survey of multiway channels in information theory: 1961–1976," *IEEE Trans. Inf. Theory*, vol. 23, no. 1, pp. 1–37, Jan. 1977.
- [5] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

- [6] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
- [7] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-Part I: System description," *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.
- [8] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-Part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, pp. 1939–1948, Nov. 2003.
- [9] A. Reznik, S. R. Kulkarni, and S. Verdú, "Degraded Gaussian multirelay channel: Capacity and optimal power allocation," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3037–3046, Dec. 2004.
- [10] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 9, pp. 3037–3063, Sep. 2005.
- [11] W. Su, A. K. Sadek, and K. J. R. Liu, "Cooperative communication protocols in wireless networks: Performance analysis and optimum power allocation," *Wireless Pers. Commun. (Springer)*, vol. 44, no. 2, pp. 181–217, Jan. 2008.
- [12] A. S. Ibrahim, A. K. Sadek, W. Su, and K. J. R. Liu, "Cooperative communications with relay-selection: When to cooperate and whom to cooperate with?," *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2814–2827, Jul. 2008.
- [13] A. K. Sadek, W. Su, and K. J. R. Liu, "Multinode cooperative communications in wireless networks," *IEEE Trans. Signal Process.*, vol. 55, no. 1, pp. 341–355, Jan. 2007.
- [14] K. J. R. Liu, A. Sadek, W. Su, and A. Kwasinski, *Cooperative Communications and Networking*. New York: Cambridge Univ. Press, 2009.
- [15] J. Jia, J. Zhang, and Q. Zhang, "Cooperative relay for cognitive radio networks," in *Proc. IEEE INFOCOM*, Apr. 2009, pp. 2304–2312.
- [16] Q. Zhang, J. Jia, and J. Zhang, "Cooperative relay to improve diversity in cognitive radio networks," *IEEE Commun. Mag.*, vol. 47, no. 2, pp. 111–117, Feb. 2009.
- [17] T. Elkourdi and O. Simeone, "Impact of secondary MAC cooperation on spectrum sharing in cognitive radio networks," in *Proc. Conf. on Inf. Sci. Syst. (CISS)*, Baltimore, MD, Mar. 2009, pp. 1–6.
- [18] D. Niyato and E. Hossain, "Market-equilibrium, competitive, and cooperative pricing for spectrum sharing in cognitive radio networks: Analysis and comparison," *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4273–4283, Nov. 2008.
- [19] M. Maskery, V. Krishnamurthy, and Q. Zhao, "Decentralized dynamic spectrum access for cognitive radios: Cooperative design of a noncooperative game," *IEEE Trans. Commun.*, vol. 57, no. 2, pp. 459–469, Feb. 2009.
- [20] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 1813–1827, May 2006.
- [21] M. Costa, "Writing on dirty paper," *IEEE Trans. Inf. Theory*, vol. 29, no. 3, pp. 439–441, May 1983.
- [22] O. Simeone, Y. Bar-Ness, and U. Spagnolini, "Stable throughput of cognitive radios with and without relaying capacity," *IEEE Trans. Commun.*, vol. 55, no. 12, pp. 2351–2360, Dec. 2007.
- [23] Y. Chen, H. Huang, Z. Zhang, P. Qiu, and V. K. N. Lau, "Cooperative spectrum access for cognitive radio network employing rateless code," in *Proc. IEEE ICC*, May 2008, pp. 326–331.
- [24] O. Simeone, I. Stanojev, S. Savazzi, Y. Bar-Ness, U. Spagnolini, and R. Pickholtz, "Spectrum leasing to cooperating secondary *ad hoc* networks," *IEEE J. Sel. Areas in Commun.*, vol. 26, no. 1, pp. 203–213, Jan. 2008.
- [25] I. Krikidis, Z. Wei, J. N. Laneman, and J. Thompson, "Cognitive legacy networks via cooperative diversity," *IEEE Commun. Lett.*, vol. 13, no. 2, pp. 106–108, Feb. 2009.
- [26] Federal Standard, 1037C, U.S. Nat. Telecommun. Inf. Administration, 1996.
- [27] D. G. Brennan, "Linear diversity combining techniques," *Proc. IEEE*, vol. 19, no. 2, pp. 331–356, Feb. 2003.



an Associate Professor with the Department of Electrical Engineering, State University of New York (SUNY) at Buffalo. From June 2002 to March 2005, he was a Postdoctoral Research Associate with the Department of Electrical and Computer Engineering and the Institute for Systems Research, University of Maryland, College Park. He coauthored the book *Cooperative Communications and Networking* (New York: Cambridge University Press, 2009).

Dr. Su is the recipient of the 2010 IEEE International Conference on Communications (ICC) Best Paper Award. He received the Invention of the Year Award from the University of Maryland in 2005. He received the Signal Processing and Communications Faculty Award from the University of Delaware in 2002. He served as an Associate Editor for the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY and the IEEE SIGNAL PROCESSING LETTERS. He also coorganized two Special Issues for IEEE JOURNALS in the field of cooperative communications and networking.



**John D. Matyjas** (M'02) received the Ph.D. degree in electrical engineering from the State University of New York, Buffalo, in 2004.

He has since been with the Air Force Research Laboratory, Rome, NY, serving as the Connectivity Core Technical Competency Lead. His research interests are in the areas of wireless multiple-access communications and networking, cognitive radio, statistical signal processing and optimization, and neural networks. Additionally, he serves as an adjunct faculty in the Department of Electrical Engineering at the State University of New York Institute of Technology, Utica/Rome.

Dr. Matyjas is the recipient of the Mohawk Valley "Engineer of the Year" Award and, as a coauthor, the 2010 IEEE International Conference on Communications (ICC) Best Paper Award. He is a member of the IEEE Communications, Information Theory, Computational Intelligence, and Signal Processing Societies; chair of the IEEE Mohawk Valley Chapter Signal Processing Society; and a member of the Tau Beta Pi and Eta Kappa Nu engineering honor societies.



**Stella Batalama** (S'91–M'94) received the Diploma degree in computer engineering and science (5-year program) from the University of Patras, Greece, in 1989 and the Ph.D. degree in electrical engineering from the University of Virginia, Charlottesville, in 1994.

In 1995, she joined the Department of Electrical Engineering, State University of New York (SUNY) at Buffalo, where she is presently a Professor. From July 2009 to June 2011, she served as the Associate Dean for Research of the School of Engineering and Applied Sciences. Since July 2010, she has been Chair of the Electrical Engineering Department. During summers 1997–2002, she was a Visiting Faculty Member with the U.S. Air Force Research Laboratory (AFRL), Rome, NY. From August 2003 to July 2004, she served as the Acting Director of the AFRL Center for Integrated Transmission and Exploitation (CITE), Rome, NY. Her research interests include small-sample-support adaptive filtering and receiver design, cooperative communications, cognitive networks, underwater communications, covert communications, steganography, compressive sampling, adaptive multiuser detection, robust spread-spectrum communications, and supervised and unsupervised optimization.

Dr. Batalama was an Associate Editor for the IEEE COMMUNICATIONS LETTERS (2000–2005) and the IEEE TRANSACTIONS ON COMMUNICATIONS (2002–2008).